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Single Spin Asymmetry In Inclusive Pion Production*

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Abstract. It is shown how the single spin asymmetry observed in inclusive pion production is related, in the helicity basis, to the imaginary part of the product of two different distribution amplitudes, rather than to the usual quark and gluon distribution functions; there is then no reason why it should be zero even in massless perturbative QCD, provided the quark intrinsic motion is taken into account. A simple model is constructed which reproduces the main features of the data.

Spin physics in large p_T inclusive hadronic processes has unique features; not only it probes the internal structure of hadrons, but, as spin dependent observables involve delicate interference effects among different amplitudes, it tests the theory at a much deeper level than unpolarized processes.

We consider here the single spin asymmetry in inclusive pion production in $p-p$ collisions, $p^\uparrow + p \to \pi + X$. Let the two protons move along the $\hat{z}$-axis in their c.m. frame and $\hat{x}$-$\hat{z}$ be the scattering plane. The proton moving in the $+\hat{z}$ direction is polarized transversely to the scattering plane, *i.e.* along ($\uparrow$) or opposite ($\downarrow$) the $\hat{y}$-axis. The single spin asymmetry A_N is then defined by:

$$A_N(x_F, p_T) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \tag{1}$$

where $d\sigma$ is the differential cross section and $\uparrow$, $\downarrow$ refer to the proton spin directions; we denote by p_L and p_T the c.m. longitudinal and transverse pion

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momentum respectively; $x_F = 2p_L/\sqrt{s}$ is the Feynman variable and $\sqrt{s}/2$ is the c.m. energy of each incident proton.

Several experimental results are available on A_N (for a list of references see [1]); the E704 Collaboration has produced the most recent, high energy ones ($\sqrt{s}/2 \simeq 10$ GeV). Two sets of measurements are relevant to our analysis:

i) $A_N(x_F, p_T)$ for $p^\uparrow + p \to \pi^\pm, \pi^0 + X$, *vs.* x_F in the p_T range $0.7 \le p_T \le 2.0$ GeV/c [2, 3]; these data show intriguing x_F dependence (Fig. 1).

ii) $A_N(x_F, p_T)$ for $p^\uparrow + p \to \pi^0 + X$, as a function of p_T (up to $p_T \simeq 4$ GeV/c), in the central region ($|x_F| \le 0.1$) [4]; in this case no p_T dependence seems to be observed and $A_N \simeq 0$ in the whole p_T range (notice that this updates and corrects some previous results of the same collaboration [5]).

A naive generalization of the QCD-factorization theorem suggests that the single spin asymmetry can be written qualitatively as:

$$A_N \sim \sum_{ab\to cd} \Delta_T G_{a/p} \otimes G_{b/p} \otimes \hat{a}_N \hat{\sigma}_{ab\to cd} \otimes D_{\pi/c} \tag{2}$$

where $G_{a/p}$ is the parton distribution function, that is the number density of partons a inside the proton, and $\Delta_T G_{a/p} = G_{a^\uparrow/p^\uparrow} - G_{a^\downarrow/p^\uparrow}$ is the difference between the number density of partons a with spin $\uparrow$ in a proton with spin $\uparrow$ and the number density of partons a with spin $\downarrow$ in a proton with spin $\uparrow$; $D_{\pi/c}$ is the number density of pions resulting from the fragmentation of parton c; $\hat{a}_N$ is the single spin asymmetry relative to the $a^\uparrow b \to cd$ elementary process and $\hat{\sigma}$ is the cross-section for such process.

The usual argument is then that the asymmetry (2) is bound to be very small because $\hat{a}_N \sim \alpha_s m_q/\sqrt{s}$ where m_q is the quark mass. This originated the widespread opinion that single spin asymmetries are essentially zero in perturbative QCD.

However, it has become increasingly clear in the last years that such conclusion need not be true because subtle spin effects might modify Eq. (2). Such modifications should take into account the parton transverse motion, higher twist contributions and possibly non perturbative effects hidden in the spin dependent distribution and fragmentation functions. Several models have been proposed which differ in practice by which part of A_N, Eq. (2), is responsible for these effects: $\Delta_T G_{a/p}$ [6, 7, 8], $\hat{\sigma}$ [9, 10] or $D_{\pi/c}$ [11, 12].

We briefly discuss here a reformulation of Eq. (2) in the helicity basis, which is more suitable for applying the factorization theorem and which allows to formulate a model for the spin dependence of the quark distributions [1]. Our approach is reminiscent of that of Ref. [6].

In the helicity basis the differential cross-section for the inclusive process $p_1(\lambda_1) + p_2(\lambda_2) \to \pi + X(\lambda_X)$ can be written in terms of helicity amplitudes as:

$$d\sigma \sim \sum_{X,\lambda_X} \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_2'} M_{\lambda_X;\lambda_1,\lambda_2}\ \rho_{\lambda_1,\lambda_2;\lambda_1',\lambda_2'}(p_1,p_2)\ M^*_{\lambda_X;\lambda_1',\lambda_2'} \tag{3}$$

where the sum over X includes also a phase space integral for the undetected particles and the matrix ρ is the helicity density matrix describing the polar-

ization state of the initial protons. In our case p_2 is unpolarized, while p_1 is transversely polarized along $\pm\hat{y}$ direction, so that Eq. (1) becomes

$$A_N = 2\,\frac{\sum_{X,\lambda_X,\lambda_2} \mathrm{Im}[M_{\lambda_X;+,\lambda_2} M^*_{\lambda_X;-,\lambda_2}]}{\sum_{X,\lambda_X,\lambda_1,\lambda_2} |M_{\lambda_X;\lambda_1,\lambda_2}|^2}\,. \tag{4}$$

Eq. (4) shows how a non zero value of A_N implies non zero interference effects between two amplitudes which only differ by one helicity index; its denominator, instead, proportional to $d\sigma^\uparrow + d\sigma^\downarrow = 2\,d\sigma^{unp}$, only depends on moduli squared of amplitudes and can be written in the parton model as:

$$d\sigma^{unp} \sim \sum_{abcd} \int dx_a dx_b\, \frac{1}{x_c}\, G_{a/p}(x_a)\, G_{b/p}(x_b)\, \frac{d\hat{\sigma}}{d\hat{t}}(ab \to cd)\, D_{\pi/c}(x_c)\,. \tag{5}$$

In order to express the numerator of Eq. (4) in terms of parton interactions we have to define $\mathcal{G}^{a/h}_{\lambda_{X_h},\lambda_a;\lambda_h}(x_a, \boldsymbol{k}_{\perp a})$ as the helicity distribution amplitude for the process $h(\lambda_h) \to a(\lambda_a) + X_h(\lambda_{X_h})$, where $\boldsymbol{k}_{\perp a}$ is the transverse momentum of the parton a inside the hadron h; these amplitudes are related to the unpolarized partonic distribution function by:

$$G_{a/p}(x_a) = \sum_{X_p,\lambda_{X_p}} \int d\boldsymbol{k}_{\perp a} \left\{ |\mathcal{G}^{a/p}_{\lambda_{X_p},+;+}(x_a, \boldsymbol{k}_{\perp a})|^2 + |\mathcal{G}^{a/p}_{\lambda_{X_p},-;+}(x_a, \boldsymbol{k}_{\perp a})|^2 \right\}\,. \tag{6}$$

By applying the same steps which lead to the partonic expression of $d\sigma^{unp}$ we get for the numerator of A_N an expression similar to Eq. (5), with $G_{a/p}(x_a)$ replaced by $\int d\boldsymbol{k}_{\perp a}\, I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a})$, where

$$I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a}) \equiv \sum_{X_p,\lambda_{X_p}} \mathrm{Im}[\mathcal{G}^{a/p}_{\lambda_{X_p},+;+}(x_a, \boldsymbol{k}_{\perp a})\, \mathcal{G}^{a/p\,*}_{\lambda_{X_p},+;-}(x_a, \boldsymbol{k}_{\perp a})]\,. \tag{7}$$

Notice that $I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a})$ has to vanish for $\boldsymbol{k}_{\perp a} = 0$, as required by helicity conservation in the forward direction; moreover, since $I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a})$ is an odd function of $\boldsymbol{k}_{\perp a}$[1], we must keep into account $\boldsymbol{k}_{\perp a}$ effects also in the partonic cross sections, otherwise we are left with $\int d\boldsymbol{k}_{\perp a}\; I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a}) = 0$. Then

$$\int d\boldsymbol{k}_{\perp a} I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a}) \frac{d\tilde{\sigma}}{d\tilde{t}}(\boldsymbol{k}_{\perp a}) = \int_{(k_{\perp a})_x > 0} d\boldsymbol{k}_{\perp a} I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a}) \left[\frac{d\tilde{\sigma}}{d\tilde{t}}(+\boldsymbol{k}_{\perp a}) - \frac{d\tilde{\sigma}}{d\tilde{t}}(-\boldsymbol{k}_{\perp a}) \right] \tag{8}$$

where $d\tilde{\sigma}/d\tilde{t}$ means that now the partonic cross section includes $\boldsymbol{k}_{\perp a}$ effects.

To give numerical estimates we need a model for the non perturbative functions $I^{a/p}_{+-}(x, \boldsymbol{k}_\perp)$; these non diagonal distribution functions play for spin

[1] This is more easily seen if we observe that our $I^{a/p}_{+-}(x_a, \boldsymbol{k}_{\perp a})$ equals $\Delta^N G_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) = \sum_{\lambda_a} \{ G_{a(\lambda_a)/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) - G_{a(\lambda_a)/p^\uparrow}(x_a, -\boldsymbol{k}_{\perp a}) \}$ defined by Sivers [6].

observables the same rôle plaid by the usual diagonal distribution functions $G_{a/p}$ in unpolarized cross-sections. We parameterize their x dependence with simple power behaviours. The dependence on $\boldsymbol{k}_\perp$ is treated, at this stage, in a simplified way: we replace the integral in Eq. (8) by the value of the integrand at some average $k_{\perp a} = \langle \boldsymbol{k}_{\perp a}^2 \rangle^{1/2}$. That is we set

$$\int d\boldsymbol{k}_{\perp a}\, I_{+-}^{a/p}(x_a, \boldsymbol{k}_{\perp a})\, \frac{d\tilde{\sigma}}{d\tilde{t}}(\boldsymbol{k}_{\perp a}) = \frac{\hat{k}_\perp}{M_h} N_a x_a^{\alpha_a} (1-x_a)^{\beta_a}\, \frac{d\tilde{\sigma}}{d\tilde{t}}(k_{\perp a}) \tag{9}$$

where M_h is some hadronic mass scale, of the order of 1 GeV, and we assume $\hat{k}_\perp \simeq 0.5$ GeV/c. N_a can be taken from the usual distribution functions [1].

Our final expression for the single spin asymmetry A_N is then

$$A_N = \frac{\sum_{abcd} \int dx_a dx_b \frac{1}{x_c} I_{+-}^{a/p}(x_a, k_{\perp a}) G^{b/p}(x_b) \left[\frac{d\tilde{\sigma}}{d\tilde{t}}(k_{\perp a}) - \frac{d\tilde{\sigma}}{d\tilde{t}}(-k_{\perp a})\right] D_{\pi/c}(x_c)}{2\sum_{abcd} \int dx_a dx_b \frac{1}{x_c} G^{a/p}(x_a) G^{b/p}(x_b) \frac{d\hat{\sigma}}{d\hat{t}}(ab \to cd) D_{\pi/c}(x_c)}\,. \tag{10}$$

In Eq. (10) we take into account, at lowest perturbative QCD order, all possible elementary interactions involving quarks and gluons. According to $SU(6)$ proton wave functions we take $I_{+-}^{u/p} > 0$ for u quarks and $I_{+-}^{d/p} < 0$ for d quarks (see footnote after Eq. (7)); the sign of $I_{+-}^{a/p}$ for the other partonic contributions is less relevant, and for the moment we assume all these contributions to be positive. However, a more careful analysis is in progress [1]. The unpolarized distribution and fragmentation functions are taken from Ref. [13, 14].

In Fig. 1 we compare our results, at $p_T = 2$ GeV/c, with the experimental data. Most contributions come from $qg \to qg$ and $gg \to gg$ processes and the parameters α and β of Eq. (9) yielding these results are $\alpha_u = \alpha_d = \alpha_g \simeq -0.6$, $\beta_u = \beta_d = 2.5$ and $\beta_g = 3.5$. We also find that, at $x_F = 0$, $A_N \simeq 0$, independently of p_T, in agreement with the most recent data [4].

Our results clearly show how a careful treatment of spin observables and the inclusion of intrinsic $k_\perp$ effects can yield sizeable values of single spin asymmetries in hadronic inclusive pion production, via perturbative QCD dynamics, contrary to widespread belief.

The off-diagonal distribution functions $I_{+-}^{a/p}$ introduced in Eq. (7) contain all the relevant non perturbative information; similarly to the parton distribution functions in unpolarized processes, they cannot be computed, but have to be taken from experiment. This is essentially what we have done here, resulting in reasonable expressions for $I_{+-}^{a/p}$; further discussions can be found in Ref. [1]. Once the non diagonal distribution functions have been obtained from one set of experiments, they can be used to make genuine perturbative QCD predictions for other spin observables like single spin asymmetries in $\pi + p^\uparrow \to \pi + X$ and $p^\uparrow + p \to \gamma + X$. Some experimental results are already available and more are soon expected.

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Figure Caption

Fig. 1 Single spin asymmetry for π^+, π^0, π^-, *vs.* x_F at $p_T = 2$ GeV/c, from Eq. (10), compared to experimental results [2, 3] (see text for details).

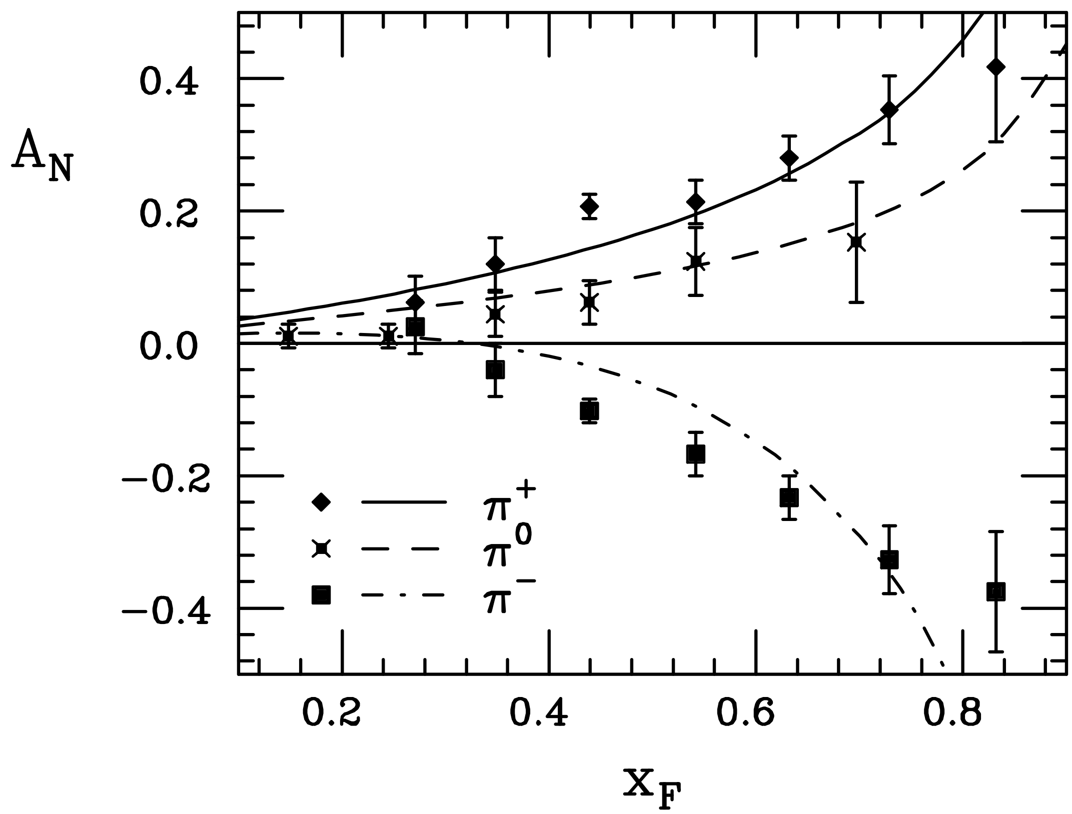

FIG. 1